Table 13.1
A summary of the type of critical point associated with the properties of the eigenvalues of the coefficient matrix of the pair of first-order differential equations.

eigenvalues	type of critical point	stability of the critical point
1. real, unequal, both negative	improper node	asymptotically stable
1. real, unequal, both positive	improper node	unstable
2. real, opposite signs	saddle	unstable
3. equal and positive	proper or improper node	unstable
3. equal and negative	proper or improper node	asymptotically stable
4. complex conjugates, real part > 0	spiral point	unstable
4. complex conjugates, real part < 0	spiral point	asymptotically stable
5. pure imaginary	center	stable

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$p = a + d = \mathrm{Tr}\, A$
$q = ad - bc = \mathrm{Det}\, A$

The eigenvalues of A are given by

$\Delta = p^2 - 4q$

$$\lambda_{\pm} = \frac{a+d}{2} \pm \frac{1}{2}[\,(a+d)^2 - 4(ad-bc)\,]^{1/2} = \frac{p}{2} \pm \frac{1}{2}[\,p^2 - 4q\,]^{1/2}$$

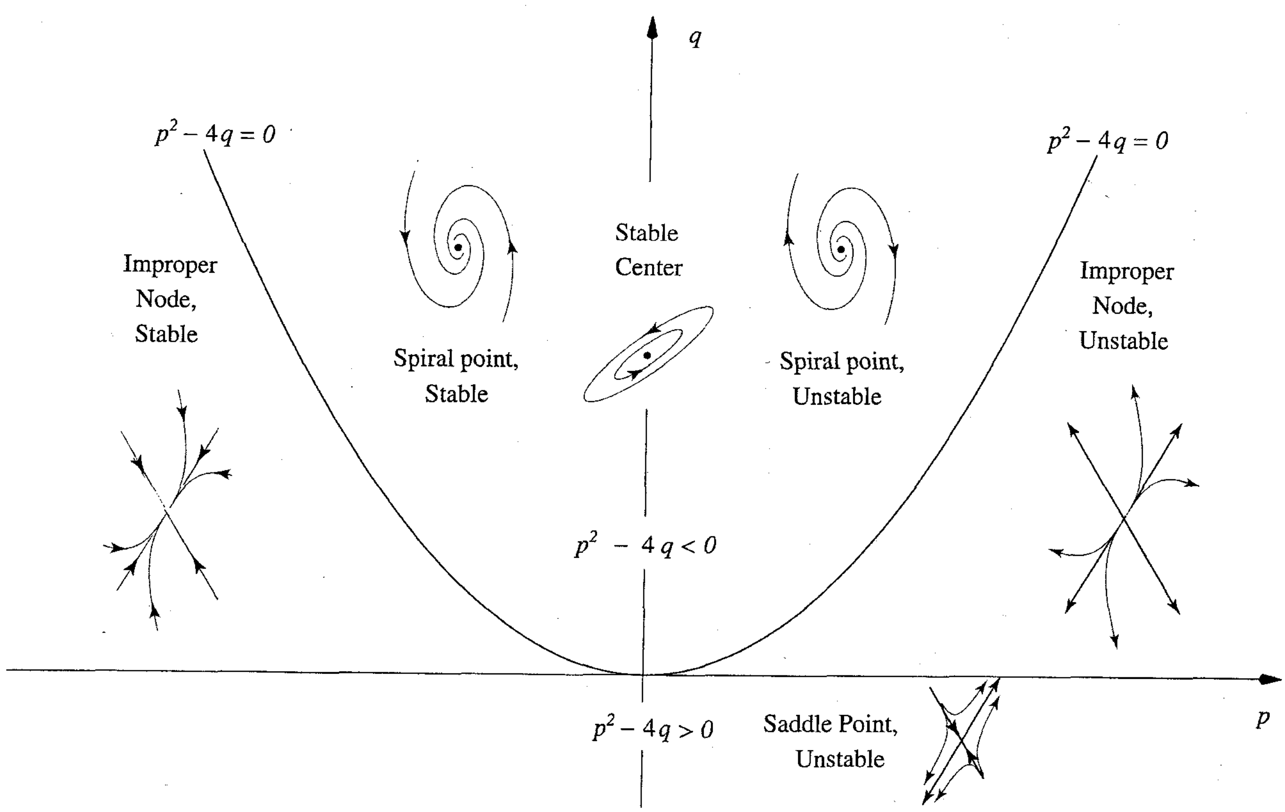

Figure 13.18
A diagram showing the various types of nodes and their stabilities as a function of $q = ad - bc$ and $p = a + d$. There is a center all along the vertical line $p = 0$ for $q > 0$ and the parabola $\Delta = p^2 - 4q = 0$ separates different types of critical points.