Fourier Analysis

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Motivation

- In science and technology, we often encounter periodic phenomena.
  - Some process repeats itself after some time interval T, called the period. f = 1/T is called the frequency.
- Periodic mathematical function expressed as

$$\varphi(t+T)=\varphi(t)$$

- Simplest periodic functions

$$A\sin(\omega t+\alpha)$$

$$A\cos(\omega t+\alpha)$$

where $\omega=\dfrac{2\pi}{T}$

is the angular frequency.

Fourier Series

- Joseph Fourier noted that almost any period function can be constructed using a combination of sine and cosine functions.
- If we add the functions

$$y_0 = A_{0,} \quad y_1 = A_1 \sin(\omega t + \alpha_1), \quad y_2 = A_2 \sin(2\omega t + \alpha_2),$$

$$y_3 = A_3 \sin(3\omega t + \alpha_3), \ldots$$

we obtain a period function with period T.

Reverse Problem

- Given an arbitrary periodic function $\varphi(t)$ can we express it as a sum of simple periodic functions?
- In almost all cases, the answer is yes, although it may require an infinite series.

$$\varphi(t) = A_0 + \sum_{n=1}^{\infty} A_0 \sin(n\omega t + \alpha_n)$$

- Each term is called a *harmonic*.

Trigonometric Fourier Expansion

- Standardized form:

$$f(x) = a_0 + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$$

- Can be extended to non-periodic functions – e.g.,

$$x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}(-1)^n \frac{\cos nx}{n^2} \quad (-\pi < x < \pi)$$