2D Heat Equation Examples

1. The initial temperature distribution within a thin uniform rectangular plate with thermally insulated lateral faces is

$$u(x, y, 0) = Axy(l_x - x)(l_y - y)$$

where A is a constant. Find the distribution of temperature within the plate at any later time if its boundary is kept at a constant zero temperature. Generation or absorption of heat by external sources is absent.

2. Find the temperature $u(x,y,t)$ of a thin rectangular plate if its boundary is kept at constant zero temperature, the initial temperature distribution within the plate is zero, and one internal source of heat with value $Q(t) = A \sin \omega t$ acts at the point (x_0, y_0). Assume the plate is thermally insulated over its lateral surfaces.

3. A heat-conducting, thin, uniform rectangular plate is thermally insulated over its lateral faces. One side of the plate, at $x = 0$, is thermally insulated and the rest of the boundary is kept at constant zero temperature. The initial temperature distribution within the plate is zero. Let heat be generated throughout the plate with the intensity of internal sources (per unit mass of the membrane) given by

$$Q(x,y,t) = A(l_x - x)\sin\frac{\pi y}{l_y}\sin t$$

Find the temperature distribution within the plate for $t > 0$.

4. A heat-conducting thin uniform rectangular plate is thermally insulated over its lateral faces. The edge at $y=0$ of the plate is kept at the constant temperature u_1; the edge $y=l_y$ is kept at the constant temperature u_2; and the remaining boundary is thermally insulated. The initial temperature distribution within the plate is $u(x,y,t) = u_0$ where u_0 is a constant. Find the temperature $u(x,y,t)$ of the plate at any later time, if generation (or absorption) of heat by internal sources is absent.

1D Wave Equation Examples

1. The ends of a uniform string of length l are fixed and all external forces including the gravitational force can be neglected. Displace the string from equilibrium by shifting the point $x = x_0$ by distance A at time $t = 0$ and then release it with zero initial speed. Find the displacements $u(x, t)$ of the string for times $t > 0$.

2. Consider a homogeneous string of mass density ρ with rigidly fixed ends. Starting at time $t = 0$ a uniformly distributed harmonic force with linear density $F(x,t) = F_0 \sin \omega t$ acts on the string. The initial deflection and speed are zero. Neglecting friction, find the resulting oscillations and investigate the resonance behavior.

3. The left end of a string is moving according to $g(t) = A \sin \omega t$ and the right end $x = l$ is secured. Initially the string is at rest. Describe oscillations when there are no external forces and the resistance of the surrounding medium is zero.