CS 4390/5390 Fall 2013
Shirley Moore, Instructor
Homework 3
Due Tuesday, October 22

You may either do the problems by hand or use a program that you have written (if appropriate). If you do the problem by hand, please show your work. If you use a program, please include the program with your homework submission and explain how you used it to solve the problem.

1. Use the appropriate theorem to compute the following:
   (a) $2^{1,000,000} \pmod{17}$
   (b) $2^{2007} \pmod{15}$
   (c) $3^{100,000} \pmod{35}$

2. For which positive integers m is each of the following statements true?
   (a) $27 \equiv 5 \pmod{m}$
   (b) $1000 \equiv 1 \pmod{m}$

3. Find the general solution of $6x + 8y = 100$.

4. Determine if each of the following linear congruences has a solution, and if so, solve it.
   (a) $8x \equiv 5 \pmod{13}$
   (b) $15x \equiv 24 \pmod{27}$
   (c) $35x \equiv 11 \pmod{49}$

5. Suppose it is known that the Diophantine equation $40x - 622y = 34$ has the complete solution $x = 203 + 311t$, $y = 13 + 20t$. What is the complete solution to the congruence $40x \equiv 34 \pmod{622}$?

6. Solve the following system of linear congruences.
   $2x \equiv 11 \pmod{23}$
   $9x \equiv 12 \pmod{33}$

7. The congruence $7^{1734250} \equiv 1660565 \pmod{1734251}$ is true. Show that 1734251 is composite.

8. (a) Find $\phi(97)$
   (b) Find $\phi(8800)$

9. Given that the only prime divisors of n = 3035888343 are 3, 19, and 47, compute $\phi(n)$.

10. Find all positive integers n such that $\phi(n) = 6$. Prove that you have found all possible solutions.

11. Show that there is no positive integer n such that $\phi(n) = 14$.

12. (a) Show that there are no positive integers n satisfying $\sigma(n) = 10$.
(b) Find the form of all positive integers n satisfying $\tau(n) = 10$. What is the smallest such integer?

13. Find a perfect number larger than 10,000.

14. Prove: If $2^n - 1$ is prime for $n > 0$, then n is prime.

15. Determine each of the following:
(a) $ord_{11}(3)$ (b) $ord_{17}(2)$ (c) $ord_{21}(10)$ (d) $ord_{25}(9)$

16. (a) How many primitive roots are there for 761?
(b) Given that 6 is the smallest primitive root for 761, find the next three primitive roots.

17. Compute $(n-1)! \pmod{n}$ for $n = 2, \ldots, 30$. Propose a theorem based on what you observe.