Please explain why it follows from $X \equiv x \pmod{b}$ and $Y \equiv y \pmod{b}$ that $X + Y \equiv x + y \pmod{b}$.

Proof:
$X \equiv x \pmod{b}$ means that X and x differ by a multiple of b, or $X = kb + x$ for some integer k. Likewise $Y = mb + y$ for some integer m. Then

$$X + Y = (kb + x) + (mb + y) = (k + m)b + (x + y).$$

That is, X+Y and x + y differ by a multiple of b, so $X + Y \equiv x + y \pmod{b}$.